TOXIN MEDIATED INTERACTIONS IN VIEW OF THE ROCK PAPER SCISSORS GAME

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Abstract. In this work, important aspects of bacteriocin producing bacteria and their interplay are elucidated (Kerr et. al., 2004). Various attempts to model the resistant, producer and sensitive *Escherichia coli* strains in the so called RPS (Rock-Paper - Scissors) game had been made in the literature (Nowak, Sigmund; 2002), (Frean, Abraham; 2001). The question arose whether there is a continuous model with a cyclic structure and admitting an oscillatory dynamics, as observed in various experiments. The traditional differential equation model of the RPS game cannot be applied either to the bacteriocin system because it involves positive interaction terms. For the first time, a continuous, spatially homogeneous model that describes the competitive interaction between bacteriocin-producing, resistant and sensitive bacteria is established (Neumann, Schuster; 2007). The interaction terms have negative coefficients. For example, in experiments with mice cultures, migration seemed to be essential for the reinfection in the RPS cycle. Often statistical and spatial effects such as migration and mutation are regarded to be essential for periodicity. Our model gives rise to oscillatory dynamics in the RPS game without such effects.

The toxicity of the bacteriocin is used as a bifurcation parameter. Exact parameter ranges are obtained for which a stable (robust) limit cycle and a stable heteroclinic cycle exist in the three-species game (Hofbauer, Siegmund; 1998). These parameters are in good accordance with the observed relations for the *E. coli* strains (Vadyvaloo et.al.; 2004). The roles of growth rate and growth yield of the three strains are discussed.

As a second major task, starting from biological ground principles (Costa et.al.; 2006), we determine evolutionary stable states or pathways in the space of traits in bacteria. We use an adaptive dynamic approach and let both the toxicity of the producer and the strategy parameter, positioning the species between high initial growth rate and high yield, evolve by random mutations. The stability type of the obtained singular point is analyzed (Geritz et.al.; 1997). The result can be generalized to all kinds of toxin or disease interactions of three species. As target dynamics we get Zeeman class 33, which is permanent and guarantees stable coexistence of three species in population dynamics and is stable with respect to adaptive evolution (Zeeman; 1993). This implies that all toxin mediated interactions tend to a stable coexistive fixed point, if reasonable biological relations between the parameters are assumed (Neumann; 2007).

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